Why do Preservice Mathematics Teachers (think they) need to study Group Theory?

Dr Olivia Fitzmaurice, University of Limerick and
Dr. Mairead Greene, Rockhurst University

Authors' Contact Information

Dr Olivia Fitzmaurice, Lecturer in Mathematics Education,
School of Education,
University of Limerick, Ireland.
Tel: + 353 61 202012
Email: Olivia.Fitzmaurice@ul.ie

Dr. Mairead Greene, Associate Professor of Mathematics
St. Ignatius Science Center 120G
Rockhurst University, 1100 Rockhurst Road, Kansas City, MO 64110.
Tel: 816-501-4530
Email: mairead.greene@rockhurst.edu

Abstract:

This study investigates the viewpoints of 20 preservice post-primary mathematics teachers taking a module of Group Theory in the final semester of their university teacher education programme. The purpose of the study was to investigate preservice mathematics teachers' perception of the value of studying a module in Group Theory and their perception of its relevance to the content they will teach at post primary school. The sample were surveyed again 12 months later, having taught at post primary level for almost a full academic year to see if their perspectives had changed. The findings are discussed in this paper.

Key Words:

Group Theory, Preservice, Mathematics, Teachers.

Introduction

Group Theory is a topic in mathematics which focuses on algebraic structures called groups. Groups are sets of objects combined with rules that govern how they can be combined. For most students this is one of the most theoretical topics that they will encounter. The purpose of this study was to investigate university preservice mathematics teachers' perception of the value of studying a module in Group Theory and their perception of its relevance to the content they will teach at post primary school. Research conducted in the area of Group Theory education is scant (Hazzan, 1999; Lewis, 2013; Suominem, 2015). The majority of papers written in this area focus either on the delivery of the content or the conceptual understanding of the participants (Hazzan, 1999; Suominem, 2015). The difficulty of the subject is generally one of the first issues mentioned for mathematics specialists and prospective mathematics teachers alike (Hazzan, 1999; Larsen, 2013; Lewis, 2013; Schubert, Gfeller, & Donohue, 2013; Suominem, 2015). Lewis (2013) posits that it is perhaps the most difficult mathematics subject at undergraduate level. One reason offered for the difficulty of the subject is the disconnect that exists between school and undergraduate mathematics. ('School mathematics' in Ireland refers to K-12 mathematics and this term will be used repeatedly throughout the paper). Secondly, students find Group Theory difficult because it is unlike any mathematics they have studied previously. It is sometimes their introduction to abstraction and formal proof, where repetition and memorisation are no longer useful tools (Capaldi, 2013; Suominem, 2015; Wheeler & Champion, 2013). Students often tend to fear proof (Barbut, 1987), find it difficult (Wheeler & Champion, 2013) and are not very good at it (Capaldi, 2013; Lewis, 2013). Consequently Group Theory is a difficult subject to teach (Capaldi, 2013).

Group Theory and Mathematics Education

In the literature available there is general consensus that the study of Group Theory is important, essential even in the study of undergraduate mathematics, yet the reasons are often vague or perhaps not specific to Group Theory (Hazzan, 1999; Suominem, 2015). Gallian (1997) believes that Group Theory is important for all who study mathematics because the applications are vast. Its relevance in areas such as computer science, physics, chemistry, data communication are evident to those who teach it and it is a necessity for anyone who wishes to pursue the study of further mathematics (Gallian, 1997; Hazzan, 1999; Lewis, 2013). Students who proceed to study advanced mathematics will be hindered if their understanding in Group Theory is compromised according to Lewis (2013).

Cullinane (2005) maintains that all prospective mathematics teachers should study Group Theory, as they will struggle to teach effectively in its absence. However Smith (2002) claims that many prospective mathematics teachers fail to see the link between Group Theory and 'school' mathematics. This disconnect means that the purpose of Group Theory is lost on the majority of participants who study it at university (Cullinane, 2005). It becomes quite a difficult subject to motivate and teach particularly to prospective teachers who do not plan to study any more mathematics after they graduate and because they do not see the relevance of Group Theory to the content they will one day teach (Barbut, 1987).

But why specifically do prospective mathematics teachers need Group Theory to teach effectively? Barbut (1987, p. 199) states that participants need to “learn the fundamentals of Group Theory including how to write simple but coherent proofs, gain an appreciation of the power and beauty of mathematical abstraction, and view this course as useful in preparation for a career as a teacher”. Gallian (1997, p. xi) believes that “students will best appreciate the abstract theory when they have a firm grasp of just what is being abstracted”. In fact the ability to ‘abstract’ is a skill in itself (Dubinsky, Dautermann, Leron, & Zazkis, 1994; Gallian, 1997; Titova, 2007). Wheeler and Champion (2013) highlight how proof is core to all mathematical studies be it school mathematics or advanced mathematics. Group Theory provides a rationale for the mechanics underpinning many ‘school’ mathematics processes Smith (2002) and enhances the cognitive development of a prospective mathematics teacher according to Dubinsky et al. (1994).

Group Theory and Irish Mathematics Education

There are three typical routes to becoming a mathematics teacher in Ireland. Firstly, as with the participants in this study, one can do a concurrent university degree which combines mathematics and mathematics education and school placement. The second route is the completion of a professional master’s in education after the completion of an honours degree with mathematics as a significant component. The third most recent route introduced is for ‘out of field’ teachers, teachers qualified in other subject areas who wish to qualify as a mathematics teacher can now do so through the completion of a 2 year, part time professional diploma which combines mathematics and mathematics education modules. The sample of participants (n=20) in this study were preservice post primary mathematics and physical education teachers who were in their final semester of a four year concurrent degree programme which comprised 11 advanced mathematics modules (including Calculus, Algebra, Statistics and Probability, Mathematics laboratory (GeoGebra, Matlab, Maple), Linear Algebra, Analysis, History of Mathematics, Differential Equations, Group Theory), two mathematics pedagogy modules and 16 weeks of mathematics teaching practice in a post primary level school, in addition to general education and physical education modules. Places on the BSc Physical Education degree are in high demand nationally so consequently entry to the programme is difficult. In Ireland, students enter post primary school at 12/13 years of age and complete 5 or 6 years of education at this level. At the end of the third year, students complete the Junior Certificate examination in 10-12 subjects. This is followed by a 2 or 3 year senior cycle which culminates in the Leaving Certificate examinations (final post primary school leaving state examination) at age 17/18. Entry to university is through a ‘Points’ system. The maximum number of points participants can achieve in the Leaving Certificate is 625 (six subjects, 100 points for a Higher Level A1, the highest grade awarded, plus there is a 25 point incentive for any student who passes Higher Level mathematics (see note at end of paper) to encourage and increase participation at this level). In 2015 participants required a minimum of 495 points (79.2% average across six Higher Level subjects) to gain entry to this BSc programme. Those who do gain entry have a choice of five electives, mathematics being the most popular one historically.

In 2010, the mathematics curriculum was changed in all post primary schools in Ireland and a new more active curriculum was introduced called Project Maths. The curriculum in Project Maths is organized around five strands: statistics and probability, geometry and trigonometry, number, algebra and functions. This new curriculum increased the emphasis on problem solving and synthesis of ideas.

The Group Theory module that underpinned this study consisted of 24 lectures and nine tutorials over a 12 week semester. The participants were required to complete five problem sheets, in groups of three, throughout the semester in their tutorials. These assignments were not included as part of the final grade but were collected and graded to gain insight into and give feedback on participants' progress. There were two mid term assessments (worth 15% each) spaced evenly throughout the semester and a final written examination worth 60%. Participants were required to submit an essay (an Application of Group Theory or 'Why a mathematics teacher should study Group Theory') for 10% of the final grade. The content includes a revision of set theory, equivalence relations, functions, binary operations, some modular arithmetic and Groups. The course exposed students to both theorems and applications. There were a number of theorems and proofs throughout the semester. Links to post primary school mathematics were made within the module in a bid to motivate the study of Group Theory to these preservice mathematics teachers who would likely never teach it.

In Ireland, Group Theory was on the post primary level mathematics syllabus up until 2010 when it was removed entirely (Post-primary in Ireland refers to the equivalent of Grades 8-12 in the US and this term will be used for the remainder of the paper). Consequently, the preservice teachers in this sample or any subsequent graduate teachers of post primary mathematics in Ireland at least will never be required to teach Group Theory to their future students. Despite this, prospective teachers of mathematics in many universities in Ireland are still required to study Group Theory as part of their teacher training. The Teaching Council of Ireland states that all accredited post primary level teachers of mathematics must have studied a minimum of 10 ECTS (European Credit Transfer System) credits in Algebra. They add that 'this must include modules in Linear Algebra, and may include modules on Group Theory (Groups, Rings, and Fields), Cryptology, Coding Theory, or Number Theory', (Teaching Council of Ireland, 2015, p. 37). The research question we sought to answer was 'Do preservice mathematics teachers perceive Group Theory to be relevant to the school mathematics they will teach when they qualify?'

Methodology

The purpose of this study was to investigate preservice mathematics teachers' perceptions of Group Theory and its relevance and usefulness to their careers. Full ethical approval was granted to proceed with the study. The authors' hypothesis is that preservice mathematics teachers do not see the relevance of Group Theory to school mathematics.

Sample

In this sample, 18 participants had studied Higher Level mathematics and two had studied Ordinary Level mathematics at post primary level. The class had a mean score of 76.8% (SD = 11.5) in their Leaving Certificate Higher Level mathematics grade. On graduation, they are fully accredited to teach post primary level mathematics in Ireland. The entire sample was approached to complete the same survey a year later, and there were 12 responses. All participants were given information sheets and consent forms and were free to opt out at any stage of the study.

Survey Development

The authors developed a survey to measure the extent to which preservice mathematics teachers find group theory useful for their teaching careers (see appendix A for sample). A slightly modified version of the survey was also created to distribute to the participants as practicing/inservice teachers. Fundamentally the surveys were the same and we will refer to them both as the survey going forward with any differences highlighted at the end of this section.

Participants were first asked to rate how useful group theory topics would be for their teaching careers. A list of forty topics covered in the module were provided ranging from set theory and functions to groups, subgroups, homomorphisms and isomorphisms and for each one participants chose whether this topic was “Not at all Relevant”, “Somewhat Relevant” or “Very Relevant” to the school mathematics that they will teach in the post primary level classroom after graduation.

Following this participants were provided with the same forty topics again but this time they were asked to specify which area of school mathematics each of these topics were relevant to. The areas that were provided were the five strands of the Irish Post primary level mathematics syllabus, entitled *Project Maths*: Probability and Statistics, Number, Algebra, Geometry and Trigonometry, and Functions. Participants also had the option to choose “None” or “Don’t Know”.

Participants were then asked to reflect on the gains that they had made in some of the overall course goals and some of the meta-learning goals that the class aimed to develop adapted from Laursen, Hassi, Kogan, Hunter, & Weston (2011) survey developed for inquiry based learning classrooms. The overall class goals and meta-goals that participants were asked to consider their progress in developing can be seen in Tables 4 and 5. In each case participants could choose from “No Gain”, “Some Gain”, or “Great Gain”.

The authors reviewed the essays submitted on the topic of “Why a mathematics teacher should study Group Theory” from the previous two offerings of this course and highlighted statements that were provided by participants to answer the question in the title. Eleven essays were analysed in total and the following (Table 1) was the result of that analysis:

Table 1: Previous responses to 'Why a mathematics teacher should study Group Theory'

Statement	**Number of Participants who Proposed a Version of this Statement out of 11 total students.**
Teachers need to have a deeper and broader knowledge base than their students and group theory contributes to this in order to become an expert teacher.	6
Group theory connects to content in the post primary level school curriculum, namely sets, functions, algebra, number theory and geometry and therefore benefits teachers when teaching these subjects.	8
Group theory provides students with opportunities to reason abstractly and move beyond imitative behavior improving their mathematical understanding. This in turn will allow them to emphasize conceptual knowledge in their future teaching and make better decisions in the classroom when responding to student work and planning curriculum.	5
Group theory allows students to develop their ability to use mathematical notation and language.	2
Group theory has applications which teachers can use in class. For example applications of group theory are found in chemistry, physics, robotics, music, elections and games.	6
Group theory allows future teachers to develop an understanding of the connections between different strands of mathematics and use this in developing student understanding in the future.	5
A knowledge of group theory would allow a future teacher to introduce something new and different to transition year students helping them to enjoy mathematics more.	2
A knowledge of group theory would allow a teacher to support past students who are studying mathematics in college.	1
Group theory is a topic which can help develop intrinsic motivation for students to learn math in future teaching.	1

The authors took the nine statements that were found from this analysis of the past essays and asked participants the level to which they agreed or disagreed with each of the statements. Participants could choose from "Strongly Disagree", "Disagree", "Undecided", "Agree", and "Strongly Agree". Finally participants were asked if they overall felt that studying group theory had benefitted them and, if so, in what way.

Data Collection and Analysis

We administered the preservice survey at the end of the semester in April 2015 (but before the final examination) to 19 of 20 participants in the class. We administered the current teacher survey 13 months later via Survey Monkey in May, 2016 and had 12 respondents. Both surveys were anonymous as no names or ID numbers were written on the pre questionnaires and for the post questionnaire, survey monkey was used and no names were used to identify the respondents. The essays which were used to construct part of the questionnaire were submitted for coursework using student ID numbers. The researcher who was not teaching on the module, was given these essays, without any identifiers to construct the questionnaire to ensure confidentiality.

For each group theory topic provided, the authors examined the percentage of participants who considered that topic “Somewhat Useful” or “Very Useful” to their future teaching of “school mathematics”. They then compared this to the percentage of participants who chose a strand of mathematics that the topic was relevant to for a cross validation of the reliability of the student responses. We also particularly noted when more than 50% of participants found that this topic was relevant to a particular strand of the “school mathematics” curriculum.

When analysing student responses to the gains they had made in achieving the course content and meta-learning goals, the authors focused on the percentage of participants who achieved Some Gains or Great Gains for each goal. The percentage of participants who “Agreed” or “Strongly Agreed” with the statements that were provided as possible justifications for the relevance of studying Group Theory to their future careers as teachers was noted and compared to the percentage of essays that had originally made those statements. Finally, the percentage of participants who overall found Group Theory useful was recorded and the reasons provided were examined.

Results

The first analysis of the data where participants were asked how relevant they perceived forty different group theory topics would be to their teaching of “school mathematics” revealed that the participants believed that the relevance of these topics varied greatly. For instance, 100% of participants perceived Set Union, Set Intersection, Functions, Injective, Surjective, Bijective, Domain, Range, and Commutativity relevant to their future teaching of “school mathematics” but only 30-40% of participants considered Lagrange's Theorem, Subgroup Theorems, Homomorphisms, Isomorphisms relevant for their future careers as teachers. The full results are shown in Table 2 below.

Table 2: Group theory topics and participants' perception of their relevance

Group Theory Topics	Percent of Participants who Found Them Somewhat Relevant or Very Relevant for School Mathematics
Set Union, Set Intersection, Functions, Injective, Surjective, Bijective, Domain, Range, Commutativity	100%
Relation, Symmetric, Associativity, Premultiplication, Postmultiplication	90% – 100%
Reflexive, Closure, Identity Element, Inverse Element, Composition, Symmetries, Proof	80% - 90%
Equivalence relation, Transitive, Binary Operations,	70% - 80%
Modular Arithmetic, Order of a Group, Permutations to represent functions, Group	60 – 70%
Order of an Element, Cayley table, Subgroups	50 – 60%
Cyclic groups, Cyclic Notation, Cosets, Normal Subgroups	40 – 50%
Lagrange's Theorem, Subgroup Theorems, Homomorphisms, Isomorphisms	30 – 40%

Some interesting results emerge when these numbers are compared with those from the second question where participants were asked to say all the strands of "school mathematics" they thought each of these topics was relevant to. Unsurprisingly for all of the topics in the 100% category above no participants choose the "None" or "Don't Know" options available. But if you examine the topics in the 30-40% category, isomorphisms for instance, only 26% of participants thought that isomorphisms were relevant to "None" of the Project Maths strands provided as compared to 63% of participants choosing that isomorphisms were "Not at All Relevant" to teaching "school mathematics" in Question 1. Another 32% of participants decided that they "Don't Know" which strand of Project Maths isomorphisms is relevant to or if it is relevant. These numbers are very similar for the other topics in the 30-40% category and this implies that there is more uncertainty around the relevance of these topics than is suggested by the data from Question 1 alone. Similarly, for the other topics the percentage of participants who answered "None" of the Project Maths strands in Question 2 is significantly lower than the percentage of participants who answered "Not at all Relevant" to school mathematics in Question 1.

The data from Question 2 also shows us that more than 90% of participants believed that Set Union and Set Intersection would be relevant to their teaching of the Number strand of Project Maths, which the topic of Sets falls under. Table 3 provided below shows the topics that more than 50% of participants considered relevant for the different strands of Project Maths. Notice that participants considered some aspect of Group Theory relevant to four of the five strands. Unsurprisingly, participants struggled to see any connections between the content of the Group Theory course and the content of the Probability and Statistics strand.

Table 3: Group theory topics and their relevance to post primary level syllabus

Strand	**Topics that more than 50% of the participants thought would be relevant for that strand.**
Functions	Relation, Equivalence Relation, Transitive, Functions, Injective, Surjective, Bijective, Domain, Range, Permutations to Represent Functions, Composition
Number	Set Union, Set Intersection, Associativity, Closure, Commutativity, Identity Element, Inverse Element
Algebra	Associativity, Closure, Commutativity, Identity Element, Inverse Element, Premultiplication, Postmultiplication, Proof
Geometry	Symmetries

The overwhelming majority of participants (89.47% and higher) believed that they made “Some Gain” or “Great Gain” in the overall course goals of the Group Theory class and the meta-learning goals of the class. The individual results can be seen in Tables 4 and 5 below.

In the final section participants were invited to leave further comments on the study of Group theory if they so wished 15 participants did and were unanimously positive in what they wrote. Seven comments mentioned that the study of Group theory had enhanced their mathematical literacy:

Developed my mathematical thinking and understanding of notation thus improving my mathematical literacy.

I think that studying group theory has given me better mathematical literacy and improved my attitudes towards proofs.

Seven participants stated that participation in the module enhanced their ability to see connections between different areas of mathematics.

Helped form connections with other maths topics and real life applications.

You realize there are greater links between topics that you didn't know before.

There were four comments that referred to the inclusion of applications within the module as a benefit.

Increased knowledge base of maths; New ways of thinking; Interesting applications I'd use in class.

By being able to see and investigate the applications from the essay was beneficial.

Table 4: Responses to: As a result of your work in your group theory classes, what GAINS did you make in your UNDERSTANDING of each of the following?

	Some Gain	Great Gain	Some/Great Gain
The main concepts explored in this class	5.26	94.74	100.00
The relationships among the concepts	21.05	78.95	100.00
Your own ways of mathematical thinking	42.11	57.89	100.00
How mathematicians think and work	57.89	31.58	89.47
How ideas from this class relate to mathematical ideas outside group theory	52.63	42.11	94.74
How ideas from this class relate to ideas outside mathematics	57.89	31.58	89.47
How to make mathematics understandable for other people	73.68	26.32	100.00

Table 5: Responses to: As a result of your work in your group theory classes, what GAINS did you make in each of the following?

	Some Gain	Great Gain	Some/Great Gain
Confidence that you can do questions and problems in group theory	5.26	94.74	100.00
Comfort in working with complex mathematical ideas	47.37	52.63	100.00
Development of a positive attitude about learning mathematics.	47.37	52.63	100.00
Confidence that you can do proofs	68.42	26.32	94.74
Appreciation of Mathematical Thinking	42.11	57.89	100.00
Comfort in communicating mathematical concepts	52.63	47.37	100.00
Confidence that you will remember what you have learned in this class	36.84	63.16	100.00
Persistence in solving problems	57.89	36.84	94.74
Comfort in teaching group theory concepts	26.32	73.68	100.00
Appreciation of different perspectives	57.89	42.11	100.00

When analysing the percentage of participants who “Agreed” or “Strongly Agreed” with the variety of statements provided as possible reasons for studying group theory we found that in general participants agreed with the statements. The one that the smallest percentage of participants agreed with was, “Group theory is a topic which can help develop intrinsic motivation for students to learn math in future teaching.” which was the statement that had shown up in only one of the eleven essays that we had analyzed. The other statement which had shown up in only one essay, “A knowledge of group theory would allow a teacher to support past students who are studying mathematics in college.” surprisingly had 89.47% of participants who agreed with it. Another interesting result was that the statement which proposed that Group Theory would help students move beyond “imitative behavior” only had 73.68% of participants who agreed with it whereas most instructors would hope that participants would not be able to resort to imitative behavior in a class at this level.

All participants determined that taking the Group Theory class had been beneficial to them as future teachers. The reasons they provided overlapped with the reasons listed in the previous question.

Follow Up Survey

A follow up survey with these participants was conducted one year later, having taught full time in a post primary level mathematics classroom. Responses were compared to investigate if this positive view of Group Theory holds. The authors acknowledge possible bias in these responses as response to the second survey was online and completely voluntary. As a result it is possible that respondents with a particular viewpoint were more likely or less likely to respond to the survey.

For the first Question all participants, in both preservice and inservice surveys, perceived the topics of set union, set intersection, functions and domain to be relevant to school mathematics. Seven topics: Equivalence Relation, Binary Operations, Inverse Elements, Order of an Element, Cyclic notation, Subgroups and Normal Subgroups, had an increase in the percentage of participants who thought they were relevant to school mathematics but these increases were marginal, ranging between 0.44% to 5.7%. The remaining 27 topics had a lower percentage of inservice teacher participants who thought they were relevant than in the previous preservice teacher survey. This decrease was large in some cases and ranged from 0.44% to 36.4%. The topics of symmetries, premultiplication, postmultiplication and composition had the largest decreases.

Table 6: Topics that more than 50% of the inservice teachers consider relevant for each strand.

Strand	**Topics that more than 50% of the inservice teachers consider relevant for each strand.**
Functions	Relation, Functions, Injective, Surjective, Bijective, Domain, Range
Number	Set Union, Set Intersection, Associativity, Closure, Commutativity, Identity Element*, Premultiplication, Proof*
Algebra	Associativity, Proof, *Functions*
Geometry	Symmetries, *Proof*
Statistics & Probability	Set union and set intersection

50% of the inservice teachers considered set union and set intersection relevant to the Statistics & Probability strand – up from 36.84% of preservice teachers. Many topics that were found to be relevant by at least 50% of the preservice teachers were now considered less so, including Equivalence Relation, Transitive, Functions, Injective, Surjective, Bijective, Domain, Range, Permutations to Represent Functions, Composition, Inverse Element, Closure, Commutativity, Identity Element, Inverse Element, Premultiplication, Postmultiplication. In general less topics were found to be relevant however they seemed to believe that proof was more relevant than before they were teaching.

The preservice teachers were less inclined to agree with all but two of the statements in Question 2 than when they were preservice teachers (see table 7). They appeared to be more in agreement that the study of Group Theory enhanced their own conceptual understanding and their teaching as a result of this. The percentage of respondents who agreed that Group Theory connects to the post primary level curriculum and therefore benefits the teachers declined

Table 7: Comparison of preservice/inservice teachers' perceptions of benefits of Group Theory

Statements about the Benefits of Group Theory	**Pre-Service % Who Agreed or Strongly Agreed with Statements**	**In-Service % Who Agreed or Strongly Agreed with Statements**
Teachers need to have a deeper and broader knowledge base than their students and group theory contributes to this in order to become an expert teacher.	100.00	75%
Group theory connects to content in the post primary level school curriculum, namely sets, functions, algebra, number theory and geometry and therefore benefits teachers when teaching these subjects.	89.47	58.33%
Group theory provides students with opportunities to reason abstractly and move beyond imitative behaviour improving their mathematical understanding. This in turn will allow them to emphasize conceptual knowledge in their future teaching and make better decisions in the classroom when responding to student work and planning curriculum.	73.68	83.33%
Group theory allows students to develop their ability to use mathematical notation and language.	94.74	83.33%
Group theory has applications which teachers can use in class. For example applications of group theory are found in chemistry, physics, robotics, music, elections and games.	89.47	66.67%
Group theory allows future teachers to develop an understanding of the connections between different strands of mathematics and use this in developing student understanding in the future.	78.95	66.67%
A knowledge of group theory would allow a future teacher to introduce something new and different to transition year students helping them to enjoy mathematics more.	78.95	83.33%
A knowledge of group theory would allow a teacher to support past students who are studying mathematics in college.	89.47	75%
Group theory is a topic which can help develop intrinsic motivation for students to learn mathematics in future teaching.	63.16	33.33%

A number of comments left by the inservice teachers gave more insight into their perceptions of Group Theory and its place on their teacher education programme. All respondents stated that the module had benefited them. One respondent stated that the links to real life examples was beneficial. Eight respondents stated how the study of Group Theory helped their own understanding of core concepts and enabled them to see links between the different strands of the curriculum.

> *'Enhanced my understanding of certain maths concepts. Topics which stand out include identity and inverse elements, binary operations and their properties, functions and composition of functions. I am therefore more confident in delivering these topics to students in a way which promotes deeper understanding'.*

> *'Enjoyed the module and is relatable to a lot of the curriculum which I teach today. Has definitely broadened my understanding of maths'.*

> *'It was actually a relevant maths module in college and I felt it did help with my understanding of mathematics instead of just doing and solving maths questions'.*

> *'Develop a greater knowledge of mathematical notation and the links that exist across the strands'.*

Others however stated that it was not relevant to what they were teaching.

> *'Teaching in England I feel there isn't as much application of group theory at times as there would be in Ireland due to different maths curriculums'.*

Two of the inservice teachers stated that while it helped enhance their own understanding of mathematics, they did not see how it would benefit their students.

> *'It challenged me and gave me a deeper mathematical understanding but I feel this will have no benefit to 99% of the students I teach. Only the 1% who need further challenge will benefit'.*

> *'It helped me understand mathematical concepts more but it doesn't directly benefit my teaching'.*

One respondent posited that the time in university would be better spent studying the post primary level curriculum rather than Group Theory.

> *'I feel this time could be better spent developing ways of teaching the leaving certificate curriculum thus developing resources for all strands and being familiar and comfortable with every topic'.*

Conclusion

The authors' hypothesis at the outset of this study was that preservice post primary level mathematics teachers do not see the relevance of Group Theory to 'school mathematics'. The literature indicates it is a difficult subject to teach and to learn because of its abstract nature and as a result can be difficult to motivate to preservice mathematics teachers who may never teach it. The findings from this survey indicate that for this sample of preservice teachers, the opposite is true. We acknowledge that one of the authors taught the module and distributed the survey to the sample and this may have influenced the answers that participants gave. That said, the surveys were anonymous and participants were encouraged to be honest and that participation would have no bearing on their final grades. While the responses indicate an awareness of the participants' ability to link the majority of the topics taught to 'school mathematics', there remains a number of areas that they fail to see any relevance to, namely homomorphisms and isomorphisms. Twelve months after completion of the initial survey, the findings from the second survey indicate that while the majority of respondents feel it helped their own understanding of core concepts, they were not as convinced of the relevance of the module to the content they were now teaching as they once were.

The Mathematical Association of America and the American Mathematical Society both recommend that mathematics modules on mathematics teacher education programmes should be designed to give special attention to topics of relevance at school level (Lester, McCormick, & Kapusuz, 2004). For it to be of any educational/mathematical value, Higher Education students who study Group Theory ought to be provided with opportunities to link the content to that which they learned at post primary level school (Smith, 2002). Making connections between content learned at Higher Education and school content will motivate preservice teachers to engage and facilitate deeper conceptual understanding (Cullinane, 2005; Smith, 2002; Suominem, 2015). The presence and purpose of Group Theory on the curriculum for preservice teachers needs to be explicit. The authors recommend to university lecturers who teach Group Theory to prospective teachers for it be taught in alignment with the school syllabus so they can see the benefit of studying it.

The alternative for participants on this particular programme is to remove it entirely and replace the module with another content module more aligned with the post primary mathematics syllabus. They currently do 3 modules in algebra, including Linear algebra which is compulsory, which already satisfies the Teaching Council of Ireland requirements for post primary teachers. This change would require getting permission from the local academic programme review committee with sufficient justification for having the module removed.

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Appendix A: Section 1 of Survey
Section 1

How relevant are the following concepts in Group Theory to the 'school maths' you will teach in your classroom?

	Not at all relevant	Somewhat relevant	Very relevant	Area(s) of school maths this concept relates to (if any)
Set Union				
Set Intersection				
Relation				
Equivalence relation				
Reflexive				
Symmetric				
Transitive				
Functions				
Injective				
Surjective				
Bijective				
Domain				
Range				
Binary operations				
Associativity				
Closure				
Commutativity				
Identity element				
Inverse element				
Modular arithmetic				
Group				
Order of a group				
Order of an element				
Cayley table				
Premultiplication				
Postmultiplication				

Permutations to represent functions				
Cyclic notation				
Composition (of permutations or transformations)				
Symmetries (reflections and rotations)				
Cyclic groups				
Subgroups				
Lagrange's theorem				
Subgroup theorems				
Centralizer				
Cosets				
Normal Subgroups				
Homomorphisms				
Isomorphisms				
Proof				